Notes on the function gsw_t_from_rho_exact(rho,SA,p)

Notes written 15th April 2011

This function, **gsw_t_from_rho_exact**(rho,SA,p) calculates the *in situ* temperature corresponding to the input values of *in situ* density, rho, Absolute Salinity, SA, and pressure, p. The function returns NaNs if

(i) the input density is too small (which would require the *in situ* temperature to exceed $40\,°\text{C}$), if

(ii) the input density exceeds the density at the temperature of maximum density (as given by **gsw_t_maxdensity_exact**(SA,p)), or if

(ii) the temperature is less than the freezing temperature as given by **gsw_t_freezing**(SA,p) (implying that we are assuming that at the freezing temperature, the seawater is saturated with air).

This function uses the full TEOS-10 Gibbs function $g\left(S_{\text{A}},t,p\right)$ of IOC *et al*. (2010), being the sum of the IAPWS-09 and IAPWS-08 Gibbs functions.

This function begins by calculating the freezing temperature, t_freezing, and the thermal expansion coefficient, **gsw_alpha_wrt_t_exact**(SA,t_freezing,p) at this temperature. If this thermal expansion coefficient is positive and exceeds $1x10^{-5}\,\text{K}^{-1}$, a modified Newton-Raphson iterative solution procedure is performed with an initial temperature value given by solving a quadratic in temperature, given the thermal expansion coefficient at the freezing temperature and the value of density at $40°\text{C}$, as given by **gsw_rho_t_exact**(SA,40,p). This quadratic is based on a Taylor series expression for density, expanded about the freezing temperature.

If the thermal expansion coefficient at the freezing temperature is less than $1x10^{-5}\,\text{K}^{-1}$ (which occurs only for Absolute Salinities less than approximately 28 g kg^{-1}, depending on pressure), the temperature of maximum density is found from **gsw_t_maxdensity_exact**(SA,p). Again a simple quadratic for temperature is solved using the density at this value of temperature and the density at $40°\text{C}$. This quadratic gives two solutions, and if the larger of the two solutions exceeds **gsw_t_maxdensity_exact**(SA,p) by more than $5°\text{C}$ there will be only one non-frozen solution and we find this solution by the modified Newton-Raphson technique.

If the larger of these two quadratic solutions exceeds **gsw_t_maxdensity_exact**(SA,p) by less than $5°\text{C}$ we avoid using the modified Newton-Raphson method and instead solve for temperature assuming that the variation of density with temperature is a quadratic function of temperature about the temperature of maximum density. This is done iteratively, with each iteration using the previous iteration to effectively estimate ρ_{TT} at the temperature of maximum density. In this part of the code, care is taken to distinguish cases where there are two valid solutions, both of which exceed the freezing temperature, from the situation where this is only one such solution.

When the modified Newton-Raphson method is used, three iterations are performed after which the density of the solution equals that of the input density to machine precision ($4.6x10^{-13}\text{ kg m}^{-3}$). When the iterative quadratic method is used, eight iterations are performed after which the density of each non-frozen solution equals that of the input density to machine precision ($2.5x10^{-13}\text{ kg m}^{-3}$).

This function **gsw_t_from_rho_exact**(rho,SA,p) is called as

```
[t,t_multiple] = gsw_t_from_rho_exact(rho,SA,p)
```

and if there is a valid second solution, it is returned as t_multiple. When there is only one solution, t_multiple is a Nan. When there are no solutions, both t and t_multiple are Nans.

References

IAPWS, 2008: Release on the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater. The International Association for the Properties of Water and Steam. Berlin, Germany, September 2008, available from www.iapws.org. This Release is referred to in the text as **IAPWS-08**.

IAPWS, 2009: Supplementary Release on a Computationally Efficient Thermodynamic Formulation for Liquid Water for Oceanographic Use. The International Association for the Properties of Water and Steam. Doorwerth, The Netherlands, September 2009, available from http://www.iapws.org. This Release is referred to in the text as **IAPWS-09**.

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org