Notes on the function gsw_sound_speed(SA,CT,p) (which is identical to gsw_sound_speed_CT(SA,CT,p))

This functions **gsw_sound_speed**(SA,CT,p) and **gsw_sound_speed_CT**(SA,CT,p) are identical functions that evaluate the sound speed c of seawater from the 48-term rational function expression for density (as described in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al*. (2010)).

Sound speed c is given in terms of the pressure derivative of *in situ* density $\hat{\rho}(S_A, \Theta, p)$ by (see sections 2.16 and 2.17 of the TEOS-10 Manual (IOC *et al*. (2010))

$$c = \left(\left. \frac{\partial \hat{\rho}}{\partial P} \right|_{S_A,\Theta} \right)^{-\frac{1}{2}} . \qquad (1)$$

Note that in order to have sound speed c in units of m s^{-1} the partial differentiation must be done with respect to pressure in Pa .

The sound speed evaluated from the 48-term rational function expression for density of McDougall *et al*. (2011) has an rms error over the oceanographic data "funnel" (see Figure 1 below) of 0.067 m s^{-1} which is almost twice the rms error of the underlying sound speed data that was incorporated into the Feistel (2008) Gibbs function, being 0.035 m s^{-1}. Hence **gsw_sound_speed**(SA,CT,p) is not quite as accurate as using the full TEOS-10 Gibbs function for evaluating sound speed in the ocean. But for dynamical oceanography where α^Θ and β^Θ are the aspects of the equation of state that, together with spatial gradients of S_A and Θ, drive ocean currents and affect the calculation of the buoyancy frequency, we may take the 48-term rational-function expression for density as essentially reflecting the full accuracy of TEOS-10. The accuracy of the 48-term rational function expression for density is illustrated as a function of pressure in Figure 2 below. The fully accurate sound speed may be evaluated from either **gsw_sound_speed_CT_exact**(SA,CT,p) or **gsw_sound_speed_t_exact**(SA,t,p).

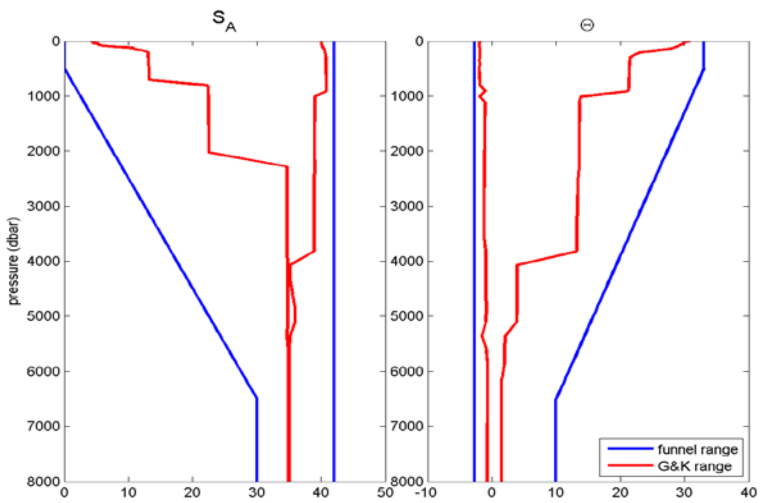

Figure 1. The ranges of Absolute Salinity and Conservative Temperature in the "oceanographic funnel" (the blue lines) in which the 48-term expression for density was fitted. The red lines shows the minimum and maximum values of Absolute Salinity and Conservative Temperature that occur in a hydrographic ocean atlas of the world ocean.

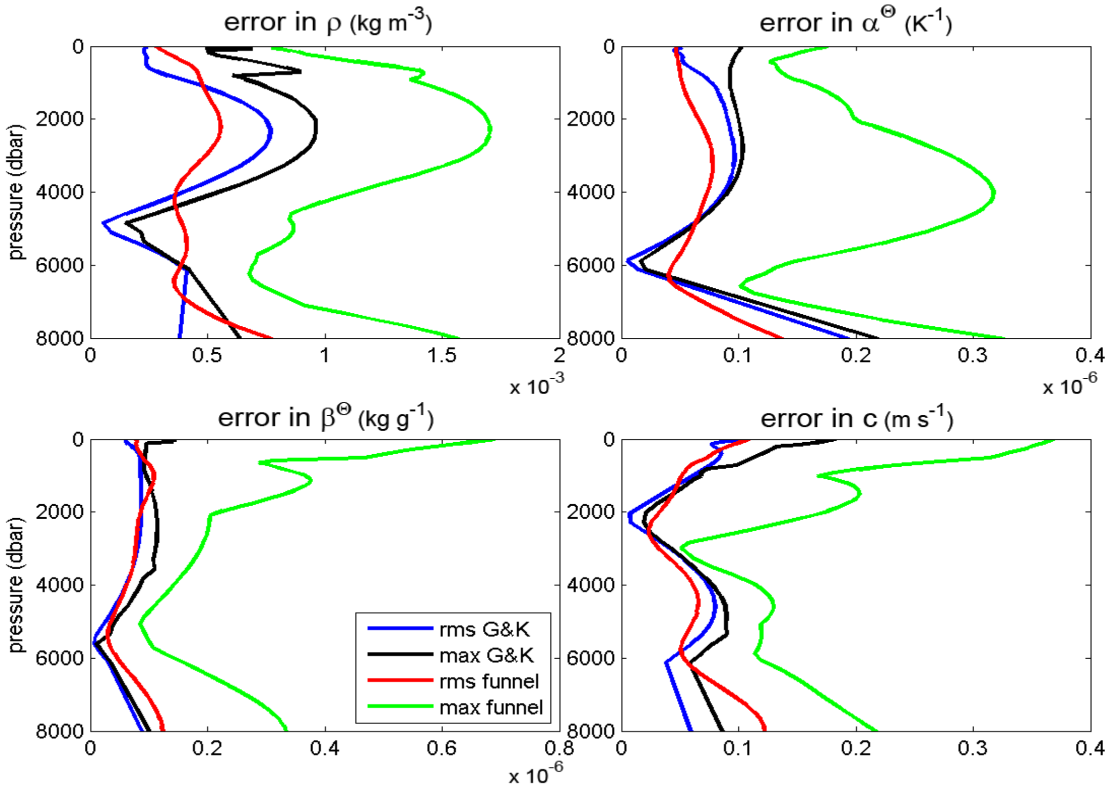

Figure 2. The errors in using the 48-term rational function expression for density, to evaluate density, the thermal expansion coefficient, the saline contraction coefficient and sound speed. The red and green lines are the r.m.s. and maximum errors for seawater in the "oceanographic funnel" (see Figure 1), while the blue and black lines are the r.m.s. and maximum errors for data in the world ocean.

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

McDougall T. J., P. M. Barker, R. Feistel and D. R. Jackett, 2011: A computationally efficient 48-term expression for the density of seawater in terms of Conservative Temperature, and related properties of seawater. submitted to *Ocean Science Discussions*.

Here follows sections 2.16 and 2.17 of the TEOS-10 Manual (IOC *et al*. (2010)).

2.16 Isentropic and isohaline compressibility

When the entropy and Absolute Salinity are held constant while the pressure is changed, the isentropic and isohaline compressibility κ is obtained:

$$\kappa = \kappa(S_A, t, p) = \rho^{-1} \frac{\partial \rho}{\partial P}\bigg|_{S_A,\eta} = -v^{-1} \frac{\partial v}{\partial P}\bigg|_{S_A,\eta} = \rho^{-1} \frac{\partial \rho}{\partial P}\bigg|_{S_A,\theta} = \rho^{-1} \frac{\partial \rho}{\partial P}\bigg|_{S_A,\Theta} = \frac{\left(g_{TP}^2 - g_{TT}\, g_{PP}\right)}{g_P\, g_{TT}} . \qquad (2.16.1)$$

The isentropic and isohaline compressibility κ is sometimes called simply the isentropic compressibility (or sometimes the "adiabatic compressibility"), on the unstated understanding that there is also no transfer of salt during the isentropic or adiabatic change in pressure. The isentropic and isohaline compressibility of seawater κ produced by both the SIA and GSW software libraries (appendices M and N) has units of Pa^{-1}.

2.17 Sound speed

The speed of sound in seawater c is given by

$$c = c(S_A, t, p) = \left(\partial P/\partial \rho\big|_{S_A,\eta}\right)^{0.5} = (\rho\kappa)^{-0.5} = g_P \left(g_{TT} \Big/ \left[g_{TP}^2 - g_{TT}\, g_{PP}\right]\right)^{0.5} . \qquad (2.17.1)$$

Note that in these expressions in Eqn. (2.17.1), since sound speed is in m s^{-1} and density has units of kg m^{-3} it follows that the pressure of the partial derivatives must be in Pa and the isentropic compressibility κ must have units of Pa^{-1}. The sound speed c produced by both the SIA and the GSW software libraries (appendices M and N) has units of m s^{-1}.