

## Notes on the function `gsw_rho_t_exact(SA,t,p)`

This function, `gsw_rho_t_exact(SA,t,p)`, evaluates the *in situ* density for given input values of Absolute Salinity  $S_A$ , *in situ* temperature  $t$ , and pressure  $p$ . This function uses the full TEOS-10 Gibbs function  $g(S_A, t, p)$  of IOC *et al.* (2010), being the sum of the IAPWS-09 and IAPWS-08 Gibbs functions. The *in situ* density is evaluated directly from the Gibbs function using Eqn. (2.8.1) of the TEOS-10 Manual (IOC *et al.*, 2010), repeated here,

$$\rho = \rho(S_A, t, p) = (g_p)^{-1} = \left( \left. \frac{\partial g}{\partial P} \right|_{S_A, T} \right)^{-1}. \quad (2.8.1)$$

### References

- IAPWS, 2008: Release on the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater. The International Association for the Properties of Water and Steam. Berlin, Germany, September 2008, available from [www.iapws.org](http://www.iapws.org). This Release is referred to in the text as **IAPWS-08**.
- IAPWS, 2009: Supplementary Release on a Computationally Efficient Thermodynamic Formulation for Liquid Water for Oceanographic Use. The International Association for the Properties of Water and Steam. Doorwerth, The Netherlands, September 2009, available from <http://www.iapws.org>. This Release is referred to in the text as **IAPWS-09**.
- IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from <http://www.TEOS-10.org>

Here follows sections 2.7 and 2.8 of the TEOS-10 Manual (IOC *et al.*, 2010).

### 2.7 Specific volume

The specific volume of seawater  $v$  is given by the pressure derivative of the Gibbs function at constant Absolute Salinity  $S_A$  and *in situ* temperature  $t$ , that is

$$v = v(S_A, t, p) = g_p = \left. \frac{\partial g}{\partial P} \right|_{S_A, T}. \quad (2.7.1)$$

Notice that specific volume is a function of Absolute Salinity  $S_A$  rather than of Reference Salinity  $S_R$  or Practical Salinity  $S_p$ . The importance of this point is discussed in section 2.8. When derivatives are taken with respect to *in situ* temperature, or at constant *in situ* temperature, the symbol  $t$  is avoided as it can be confused with the same symbol for time. Rather, we use  $T$  in place of  $t$  in the expressions for these derivatives.

For many theoretical and modeling purposes in oceanography it is convenient to regard the independent temperature variable to be Conservative Temperature  $\Theta$  rather than *in situ* temperature  $t$ . We note here that the specific volume is equal to the pressure derivative of specific enthalpy at fixed Absolute Salinity when any one of  $\eta$ ,  $\theta$  or  $\Theta$  is also held constant, as follows (from appendix A.11)

$$\left. \frac{\partial h}{\partial P} \right|_{S_A, \eta} = \left. \frac{\partial h}{\partial P} \right|_{S_A, \theta} = \left. \frac{\partial h}{\partial P} \right|_{S_A, \Theta} = v. \quad (2.7.2)$$

The use of  $P$  in these equations emphasizes that it must be in Pa not dbar. Specific volume  $v$  has units of  $\text{m}^3 \text{kg}^{-1}$  in both the SIA and GSW software libraries.

## 2.8 Density

The density of seawater  $\rho$  is the reciprocal of the specific volume. It is given by the reciprocal of the pressure derivative of the Gibbs function at constant Absolute Salinity  $S_A$  and *in situ* temperature  $t$ , that is

$$\rho = \rho(S_A, t, p) = (g_p)^{-1} = \left( \partial g / \partial P \Big|_{S_A, T} \right)^{-1}. \quad (2.8.1)$$

Notice that density is a function of Absolute Salinity  $S_A$  rather than of Reference Salinity  $S_R$  or Practical Salinity  $S_p$ . This is an extremely important point because Absolute Salinity  $S_A$  in units of  $\text{g kg}^{-1}$  is numerically greater than Practical Salinity by between  $0.165 \text{ g kg}^{-1}$  and  $0.195 \text{ g kg}^{-1}$  in the open ocean so that if Practical Salinity were inadvertently used as the salinity argument for the density algorithm, a significant density error of between  $0.12 \text{ kg m}^{-3}$  and  $0.15 \text{ kg m}^{-3}$  would result.

For many theoretical and modeling purposes in oceanography it is convenient to regard density to be a function of Conservative Temperature  $\Theta$  rather than of *in situ* temperature  $t$ . That is, it is convenient to form the following two functional forms of density,

$$\rho = \hat{\rho}(S_A, \Theta, p), \quad (2.8.2)$$

where  $\Theta$  is Conservative Temperature. We will adopt the convention (see Table L.2 in appendix L) that when enthalpy  $h$ , specific volume  $v$  or density  $\rho$  are taken to be functions of potential temperature they attract an over-tilde as in  $\tilde{v}$  or  $\tilde{\rho}$ , and when they are taken to be functions of Conservative Temperature they attract a caret as in  $\hat{v}$  and  $\hat{\rho}$ . With this convention, expressions involving partial derivatives such as (2.7.2) can be written more compactly as (from appendix A.11)

$$\hat{h}_p = \tilde{h}_p = \hat{h}_p = v = \rho^{-1} \quad (2.8.3)$$

since the other variables are taken to be constant during the partial differentiation. Appendix P lists expressions for many thermodynamic variables in terms of the thermodynamic potentials

$$h = \hat{h}(S_A, \eta, p), \quad h = \tilde{h}(S_A, \theta, p) \quad \text{and} \quad h = \hat{h}(S_A, \Theta, p). \quad (2.8.4)$$

Density  $\rho$  has units of  $\text{kg m}^{-3}$  in both the SIA and GSW software libraries.

Computationally efficient expressions for  $\hat{\rho}(S_A, \Theta, p)$  and  $\tilde{\rho}(S_A, \theta, p)$  involving 48 coefficients are available (McDougall *et al.* (2011b)) and are described in appendix A.30 and appendix K. These expressions can be integrated with respect to pressure to provide closed expressions for  $\hat{h}(S_A, \Theta, p)$  and  $\tilde{h}(S_A, \theta, p)$  (see Eqn. (A.30.6)).