Notes on the function gsw_isopycnal_vs_ntp_CT_ratio(SA,CT,p)

This function **gsw_isopycnal_vs_ntp_CT_ratio**(SA,CT,p) evaluates the "isopycnal temperature gradient ratio" defined by (from section 3.17 of the TEOS-10 Manual, IOC *et al*. (2010))

$$G^{\Theta} \;\equiv\; \frac{\left[R_{\rho}-1\right]}{\left[R_{\rho}/r-1\right]} \;. \tag{3.17.4}$$

This is the ratio of the (parallel) gradient of Conservative Temperature in a potential density surface, $\nabla_{\sigma}\Theta$, to that in a neutral tangent plane, $\nabla_{n}\Theta$, since, from Eqn. (3.17.3) of the TEOS-10 Manual,

$$\nabla_{\sigma}\Theta \;=\; \frac{r\left[R_{\rho}-1\right]}{\left[R_{\rho}-r\right]}\nabla_{n}\Theta \;=\; G^{\Theta}\nabla_{n}\Theta \;. \tag{3.17.3}$$

This function, **gsw_isopycnal_vs_ntp_CT_ratio**(SA,CT,p), uses the 48-term expression for density, $\hat{\rho}\left(S_{\mathrm{A}},\Theta,p\right)$. This 48-term rational function expression for density is discussed in McDougall *et al*. (2011) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al*. (2010)). For dynamical oceanography we may take the 48-term rational function expression for density as essentially reflecting the full accuracy of TEOS-10.

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

McDougall T. J., P. M. Barker, R. Feistel and D. R. Jackett, 2011: A computationally efficient 48-term expression for the density of seawater in terms of Conservative Temperature, and related properties of seawater. submitted to *Ocean Science Discussions*.

Here follows section 3.17 of the TEOS-10 Manual (IOC *et al*. (2010)).

3.17 Property gradients along potential density surfaces

The two-dimensional gradient of a scalar φ along a potential density surface $\nabla_{\sigma}\varphi$ is related to the corresponding gradient in the neutral tangent plane $\nabla_{n}\varphi$ by

$$\nabla_{\sigma}\varphi \;=\; \nabla_{n}\varphi + \frac{\varphi_{z}}{\Theta_{z}}\frac{R_{\rho}\left[r-1\right]}{\left[R_{\rho}-r\right]}\nabla_{n}\Theta \tag{3.17.1}$$

(from McDougall (1987a)), where r is the ratio of the slope on the $S_{\mathrm{A}}-\Theta$ diagram of an isoline of potential density with reference pressure p_{r} to the slope of a potential density surface with reference pressure p, and is defined by

$$r \;=\; \frac{\alpha^{\Theta}\left(S_{\mathrm{A}},\Theta,p\right)/\beta^{\Theta}\left(S_{\mathrm{A}},\Theta,p\right)}{\alpha^{\Theta}\left(S_{\mathrm{A}},\Theta,p_{\mathrm{r}}\right)/\beta^{\Theta}\left(S_{\mathrm{A}},\Theta,p_{\mathrm{r}}\right)} \;. \tag{3.17.2}$$

Substituting $\varphi=\Theta$ into (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of Θ

$$\nabla_\sigma \Theta = \frac{r\left[R_\rho - 1\right]}{\left[R_\rho - r\right]} \nabla_n \Theta = G^\Theta \nabla_n \Theta \qquad (3.17.3)$$

where the "isopycnal temperature gradient ratio"

$$G^\Theta \equiv \frac{\left[R_\rho - 1\right]}{\left[R_\rho / r - 1\right]} \qquad (3.17.4)$$

has been defined as a shorthand expression for future use. This ratio G^Θ is available in the GSW Toolbox from the algorithm **gsw_isopycnal_vs_ntp_CT_ratio**, while the ratio r of Eqn. (3.17.2) is available there as **gsw_isopycnal_slope_ratio**. Substituting $\varphi = S_A$ into Eqn. (3.17.1) gives the following relation between the (parallel) isopycnal and epineutral gradients of S_A

$$\nabla_\sigma S_A = \frac{\left[R_\rho - 1\right]}{\left[R_\rho - r\right]} \nabla_n S_A = \frac{G^\Theta}{r} \nabla_n S_A . \qquad (3.17.5)$$