Notes on the function

gsw_geo_strf_isopycnal(SA,CT,p,p_ref,Neutral_Density,p_Neutral_Density,A)

Notes written 23rd September 2010, revised 19th April 2011

gsw_geo_strf_isopycnal(SA,CT,p,p_ref,Neutral_Density,p_Neutral_Density,A) evaluates the "isopycnal" geostrophic streamfunction of McDougall and Klocker (2010), φ^{n}, relative to a reference pressure p_ref. It uses the computationally-efficient 48-term rational function expression of McDougall *et al*. (2011) for the specific volume of seawater $\hat{v}(S_{\mathrm{A}},\Theta,p)$ in terms of Absolute Salinity S_{A}, Conservative Temperature Θ and pressure p. This 48-term rational function expression for density is discussed in McDougall *et al*. (2011) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al*. (2010)). For dynamical oceanography we may take the 48-term rational function expression for density as essentially reflecting the full accuracy of TEOS-10.

The input variables SA,CT,p are either a single vertical cast or a series of such vertical casts. The input variable p_{ref} is a single positive scalar reference pressure (in dbar). When p_ref is zero, **gsw_geo_strf_isopycnal** returns the "isopycnal" geostrophic streamfunction with respect to the sea surface, otherwise, the function returns the isopycnal geostrophic streamfunction with respect to the (deep) reference pressure p_ref.

The specific volume anomaly we use, $\tilde{\tilde{\delta}}(S_{\mathrm{A}},\Theta,p)$, is defined with respect to the specially selected values $\tilde{\tilde{S}}_{\mathrm{A}}$ and $\tilde{\tilde{\Theta}}$ which are constant on each "isopycnal", and the specific volume anomaly $\tilde{\tilde{\delta}}(S_{\mathrm{A}},\Theta,p)$ is

$$\tilde{\tilde{\delta}}(S_{\mathrm{A}},\Theta,p) = \hat{v}(S_{\mathrm{A}},\Theta,p) - \hat{v}\big(\tilde{\tilde{S}}_{\mathrm{A}},\tilde{\tilde{\Theta}},p\big). \tag{1}$$

The approximate "isopycnal" geostrophic streamfunction of McDougall and Klocker (2010) is defined by Eqn. (3.30.1) of the TEOS-10 Manual (IOC *et al*., 2010) in terms of the reference "bottle" $(\tilde{\tilde{S}}_{\mathrm{A}},\tilde{\tilde{\Theta}},\tilde{\tilde{p}})$ as follows

$$\varphi^{\mathrm{n}} = \tfrac{1}{2}\big(P-\tilde{\tilde{P}}\big)\tilde{\tilde{\delta}}(S_{\mathrm{A}},\Theta,p) - \tfrac{1}{12}\rho^{-1}T_{\mathrm{b}}^{\Theta}\big(\Theta-\tilde{\tilde{\Theta}}\big)\big(P-\tilde{\tilde{P}}\big)^{2} - \int_{P_0}^{P}\tilde{\tilde{\delta}}\,dP', \tag{2}$$

where $\rho^{-1}T_{\mathrm{b}}^{\Theta}$ is taken to be the constant value $2.7x10^{-15}\,\mathrm{K}^{-1}(\mathrm{Pa})^{-2}\mathrm{m}^{2}\mathrm{s}^{-2}$. The last term in Eqn. (2) can be expressed in terms of the dynamic height anomaly Ψ (which is defined in terms of $S_{\mathrm{SO}} \equiv 35.165\,04\ \mathrm{g\ kg}^{-1}$ and $\Theta = 0°\mathrm{C}$) as follows

$$-\int_{P_0}^{P}\tilde{\tilde{\delta}}\,dP' = \Psi + \hat{h}\big(\tilde{\tilde{S}}_{\mathrm{A}},\tilde{\tilde{\Theta}},p\big) - c_p^0\tilde{\tilde{\Theta}} - \hat{h}\big(S_{\mathrm{SO}},\Theta=0°\mathrm{C},p\big), \tag{3}$$

where the second term in Eqn. (3) is obtained from a call to **gsw_enthalpy** and the last term via a call to **gsw_enthalpy_SSO_0_p**(p) which is simply a streamlined version of **gsw_enthalpy** which takes advantage of the Conservative Temperature being zero.

Combining equations (2) and (3) we have

$$\begin{aligned}\varphi^{\mathrm{n}} = {} & \tfrac{1}{2}\big(P-\tilde{\tilde{P}}\big)\tilde{\tilde{\delta}}(S_{\mathrm{A}},\Theta,p) - \tfrac{1}{12}\rho^{-1}T_{\mathrm{b}}^{\Theta}\big(\Theta-\tilde{\tilde{\Theta}}\big)\big(P-\tilde{\tilde{P}}\big)^{2} + \Psi \\ & + \hat{h}\big(\tilde{\tilde{S}}_{\mathrm{A}},\tilde{\tilde{\Theta}},p\big) - c_p^0\tilde{\tilde{\Theta}} - \hat{h}\big(S_{\mathrm{SO}},\Theta=0°\mathrm{C},p\big),\end{aligned} \tag{4}$$

and this is how the **gsw_geo_strf_isopycnal** code evaluates the "isopycnal" geostrophic streamfunction. The dynamic height anomaly Ψ is found from the GSW function **gsw_geo_strf_dyn_height**(SA,CT,p,p_ref).

When p_ref is set to 0 dbar both Ψ and φ^{n} are the respective geostrophic streamfunctions relative to the sea surface, and when p_ref is set to a (deep) reference pressure, p_ref, both Ψ and φ^{n} are the respective geostrophic streamfunctions relative to this reference pressure surface.

The approximate isopycnal geostrophic streamfunction of McDougall and Klocker (2010) is for use in approximately neutral surfaces, such as an ω-surface of Klocker *et al.* (2009a,b) or a Neutral Density (γ^{n}) surface of Jackett and McDougall (1997)) or a suitably referenced potential density surface such as a σ_2 surface.

In this function, **gsw_geo_strf_isopycnal**, the reference values $\left(\tilde{\tilde{S}}_{\mathrm{A}}, \tilde{\tilde{\Theta}}, \tilde{\tilde{p}}\right)$ are found by interpolation down a single reference cast which includes these three variables as well as values of Neutral Density γ^{n} and also of σ_2. This reference cast is interpolated with respect to γ^{n}, to the value of γ^{n} which is supplied by the user of this function, with one value of γ^{n} for each "isopycnal" surface that is selected. If this function is being used with a series of approximately neutral surfaces which are neither Neutral Density surfaces nor ω-surfaces, then a value of γ^{n} needs to be assigned to each of the user's surfaces. This could be achieved, for example, by the user labeling a vertical profile of her/his data from the mid equatorial Pacific with Neutral Density and associating these values of Neutral Density to the whole of the user's surfaces; with one value of γ^{n} for each selected approximately neutral surface. As an alternative to having to label each layer with Neutral Density, we have also allowed for the option of having the reference data set to be interpolated with respect to σ_2, the potential density with respect to 2000 dbar. In this case, the user needs to supply the σ_2 value for each layer on which the approximate "isopycnal" streamfunction is required.

In the call to this function, **gsw_geo_strf_isopycnal**, the input variables are (SA,CT,p,p_ref,Neutral_Density,p_Neutral_Density,A), and the user supplies the Absolute Salinity SA, Conservative Temperature CT and pressure p down a vertical profile (or many such vertical profiles). The user requests the approximate "isopycnal" geostrophic streamfunction of McDougall-Klocker (2010) on a series of density surfaces (specified in the list Neutral_Density) which intersect the successive vertical profiles at p_Neutral_Density. The last argument of the calling sequence to this function, A, must be either 'gn' or 's2' which tells the code whether the interpolating variable for the reference cast is γ^{n} or σ_2.

Figure 1 below (from McDougall and Klocker (2010)) illustrates the errors in determining geostrophic velocities from various geostrophic streamfunctions in an ω-surface. The surface is global in extent and has an average pressure of 1300 dbar. The McDougall-Klocker geostrophic streamfunction φ^{n} is labeled "Eqn (62)" in Figure 1(d).

The reference values $\left(\tilde{\tilde{S}}_{\mathrm{A}}, \tilde{\tilde{\Theta}}, \tilde{\tilde{p}}\right)$ in this function, **gsw_geo_strf_isopycnal**, are taken from the equatorial Pacific Ocean and have been chosen so that the geostrophic streamfunction can be used throughout the whole world ocean. If one's study area was restricted to a single ocean basin, higher accuracy could be achieved by forming the reference values as the average Absolute Salinity, Conservative Temperature and pressure on each approximately neutral surface in one's area of interest. However, once the study area embraces both northern and southern hemisphere data of any ocean (Atlantic, Indian or Pacific), this averaging approach is not recommended, and the reference values of the standard function should be used.

The two-dimensional gradient of φ^{n} in an approximately neutral surface (ans) is simply related to the difference between the horizontal geostrophic velocity $\mathbf{v}$ in this surface and that at the reference pressure (p_ref), $\mathbf{v}_{\mathrm{ref}}$, according to

$$\mathbf{k} \times \nabla_{\mathrm{ans}} \varphi^{\mathrm{n}} = f\mathbf{v} - f\mathbf{v}_{\mathrm{ref}} \qquad \text{or} \qquad \nabla_{\mathrm{ans}} \varphi^{\mathrm{n}} = -\mathbf{k} \times \left(f\mathbf{v} - f\mathbf{v}_{\mathrm{ref}} \right). \tag{5}$$

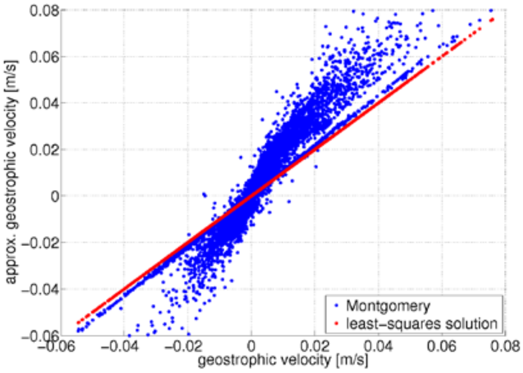

Figure 1(a)

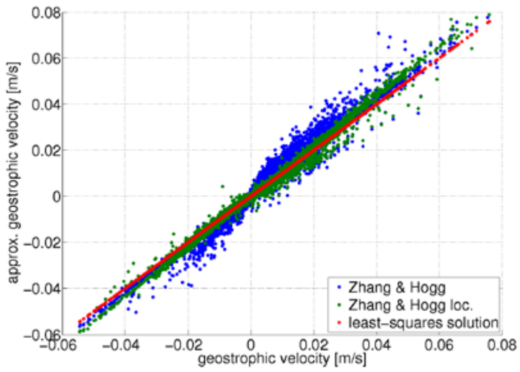

Figure 1(b)

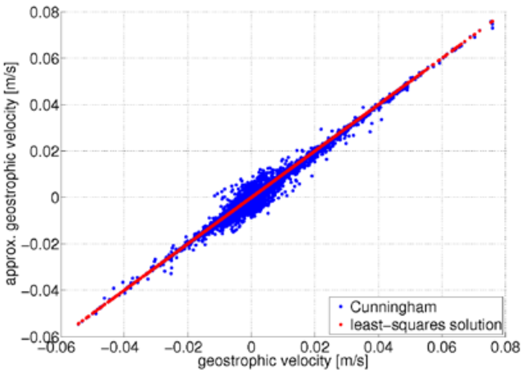

Figure 1(c)

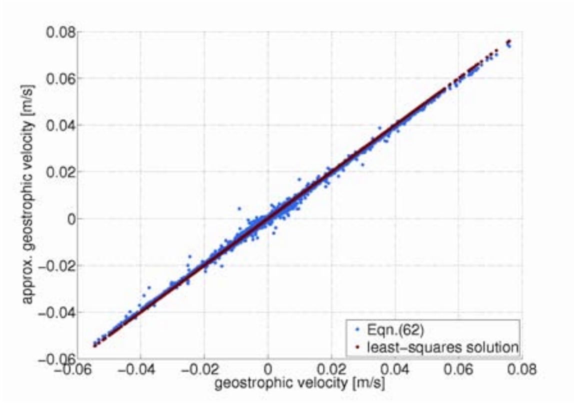

Figure 1(d)

Figure 1. Geostrophic velocities vs. approximate geostrophic velocities calculated from approximate geostrophic streamfunctions (all relative to the geostrophic velocity on the isobaric surface p = 2000 dbar) for (a) the Montgomery streamfunction Ψ^{M}, (b) the Zhang and Hogg (1992) modification to the Montgomery streamfunction, $\Psi^{\text{Z-H}}$, as well as the localized form of this streamfunction, $\tilde{\tilde{\Psi}}^{\text{Z-H}}$, (c) the Cunningham streamfunction Π, and (d) the streamfunction φ^{n}. The geostrophic velocities are evaluated on an ω-surface with an average pressure of 1300 dbar. The red dots labeled "least-squares solution" are the correct velocities.

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org See sections 3.27, 3.32 and appendix A.30.

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McDougall T. J., P. M. Barker, R. Feistel and D. R. Jackett, 2011: A computationally efficient 48-term expression for the density of seawater in terms of Conservative Temperature, and related properties of seawater. submitted to *Ocean Science Discussions*.

Here follows section 3.30 of the TEOS-10 Manual (IOC *et al*. (2010)).

3.30 Geostrophic streamfunction in an approximately neutral surface

In order to evaluate a relatively accurate expression for the geostrophic streamfunction in an approximately neutral surface a suitable reference seawater parcel $\left(\tilde{\tilde{S}}_A, \tilde{\tilde{\Theta}}, \tilde{\tilde{p}}\right)$ is selected from the approximately neutral surface that one is considering, and the specific volume anomaly $\tilde{\tilde{\delta}}$ is defined as in (3.7.3) above. The approximate geostrophic streamfunction φ^{n} is given by (from McDougall and Klocker (2010))

$$\varphi^{\mathrm{n}} \;=\; \tfrac{1}{2}\left(P-\tilde{\tilde{P}}\right)\tilde{\tilde{\delta}}\left(S_A,\Theta,p\right) \;-\; \tfrac{1}{12}\rho^{-1}T_{\mathrm{b}}^{\Theta}\left(\Theta-\tilde{\tilde{\Theta}}\right)\left(P-\tilde{\tilde{P}}\right)^2 \;-\; \int_{P_0}^{P}\tilde{\tilde{\delta}}\,dP' . \qquad (3.30.1)$$

This expression is more accurate than the Montgomery and Cunningham geostrophic streamfunctions when used in potential density surfaces, in the ω-surfaces of Klocker *et al.* (2009a,b) and in the Neutral Density surfaces of Jackett and McDougall (1997). That is, in these surfaces $\nabla_n\varphi^{\mathrm{n}} \approx \frac{1}{\rho}\nabla_z P - \nabla\Phi_0 = -\mathbf{k}\times\left(f\mathbf{v} - f\mathbf{v}_0\right)$ to a very good approximation. In Eqn. (3.30.1) $\rho^{-1}T_{\mathrm{b}}^{\Theta}$ is taken to be the constant value $2.7x10^{-15}\mathrm{K}^{-1}(\mathrm{Pa})^{-2}\mathrm{m}^2\mathrm{s}^{-2}$. This approximate isopycnal geostrophic streamfunction of McDougall and Klocker (2010) is available as the function **gsw_geo_strf_isopycnal** in the GSW Toolbox. When the last argument of this function, p_ref, is other than zero, the function returns the isopycnal geostrophic streamfunction with respect to a (deep) reference sea pressure p_ref, rather than with respect to the sea surface at $p = 0$ dbar (i.e. $P = P_0$) as in Eqn. (3.30.1).