Notes on the function, gsw_brineSA_t(t,p,saturation_fraction), which evaluates the Absolute Salinity at which seawater freezes for given *in situ* temperature, pressure and saturation_fraction of dissolved air

This function, **gsw_brineSA_t**, finds the Absolute Salinity S_A (in g kg^{-1}) at which seawater freezes for given values of *in situ* temperature t (ITS-90 °C) and pressure p (dbar). The third argument is optional and is the saturation fraction (between 0 and 1) of dissolved air in seawater. If this third argument is missing, the seawater is taken to be saturated with dissolved air (i. e. saturation_fraction is put equal to 1).

This function uses a modified Newton-Raphson iterative solution technique to solve the equation

$$t = \textbf{gsw\_t\_freezing}(\text{SA, p, saturation\_fraction}), \tag{1}$$

for Absolute Salinity SA. The initial value of SA is obtained from a polynomial and the derivative $\partial t/\partial S_A\big|_p$ is found by differentiating a polynomial expression for the in situ freezing temperature as a function of (SA, p, saturation_fraction); this polynomial can also be found in the Matlab GSW code of the **gsw_t_freezing** function, where it is commented out in favour of the more accurate calculation.

Negative values of Absolute Salinity are avoided during the calculations of **gsw_brineSA_t** by first calculating the *in situ* temperature at which pure water freezes, namely $t_0^{\text{freeze}} \equiv$ **gsw_t_freezing**(0, p, saturation_fraction). Any data points having t greater than this value are not processed through the function, thus avoiding taking fractional powers of negative numbers. Also, when the initial polynomial produces the first estimate of Absolute Salinity as being less than 2.5 g kg^{-1}, this initial estimate is replaced by one proportional to $\left(t_0^{\text{freeze}} - t\right)$. The combination of these two procedures means that negative Absolute Salinities are never processed through the function.

Five iterations of the modified Newton-Raphson iteration are performed and Eqn. (1) above is then satisfied to within machine precision of $\sim 2x10^{-14}$ °C. The accuracy and the range of applicability of the forward function **gsw_t_freezing** is described in the notes to that function in this GSW Oceanographic Toolbox.