Notes on the function gsw_Turner_Rsubrho(SA, CT, p) which evaluates the Turner angle and the Stability Ratio

This function, **gsw_Turner_Rsubrho**(SA,CT,p), evaluates the Turner angle Tu and the Stability Ratio R_ρ of the water column using the 48-term expression for density, $\hat{\rho}(S_A, \Theta, p)$. This 48-term rational function expression for density is discussed in McDougall *et al*. (2011) and in appendix A.30 and appendix K of the TEOS-10 Manual (IOC *et al*. (2010)). For dynamical oceanography we may take the 48-term rational function expression for density as essentially reflecting the full accuracy of TEOS-10.

This function **gsw_Turner_Rsubrho**(SA,CT,p) evaluates the expressions in Eqns. (3.15.1) and (3.16.1) of the TEOS-10 Manual (IOC *et al*. (2010)), and specifically, the expressions based on gradients of Conservative Temperature (rather than of potential temperature).

References

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org

McDougall T. J., P. M. Barker, R. Feistel and D. R. Jackett, 2011: A computationally efficient 48-term expression for the density of seawater in terms of Conservative Temperature, and related properties of seawater. submitted to *Ocean Science Discussions*.

McDougall, T. J., S. A. Thorpe and C. H. Gibson, 1988: Small-scale turbulence and mixing in the ocean: A glossary, in *Small-scale turbulence and mixing in the ocean*, edited by J. C. J. Nihoul and B. M. Jamart, Elsevier, Amsterdam. 3-9.

Here follows sections 3.15 and 3.16 of the TEOS-10 Manual (IOC *et al*. (2010)).

3.15 Stability ratio

The stability ratio R_ρ is the ratio of the vertical contribution from Conservative Temperature to that from Absolute Salinity to the static stability N^2 of the water column. From (3.10.1) above we find

$$R_\rho = \frac{\alpha^\Theta \Theta_z}{\beta^\Theta (S_A)_z} \approx \frac{\alpha^\theta \theta_z}{\beta^\theta (S_A)_z} . \qquad (3.15.1)$$

The stability ratio R_ρ is available in the GSW Oceanographic Toolbox from the function **gsw_Turner_Rsubrho**.

3.16 Turner angle

The Turner angle Tu, named after J. Stewart Turner, is defined as the four-quadrant arctangent (Ruddick (1983) and McDougall *et al.* (1988), particularly their Figure 1)

$$\begin{aligned} Tu &= \tan^{-1}\left(\alpha^{\Theta}\Theta_z + \beta^{\Theta}\left(S_A\right)_z,\ \alpha^{\Theta}\Theta_z - \beta^{\Theta}\left(S_A\right)_z\right) \\ &\approx \tan^{-1}\left(\alpha^{\theta}\theta_z + \beta^{\theta}\left(S_A\right)_z,\ \alpha^{\theta}\theta_z - \beta^{\theta}\left(S_A\right)_z\right) \end{aligned} \qquad (3.16.1)$$

where the first of the two arguments of the arctangent function is the "y"-argument and the second one the "x"-argument, this being the common order of these arguments in Fortran and MATLAB. The Turner angle Tu is quoted in degrees of rotation. Turner angles between 45° and 90° represent the "salt-finger" regime of double-diffusive convection, with the strongest activity near 90°. Turner angles between −45° and −90° represent the "diffusive" regime of double-diffusive convection, with the strongest activity near −90°. Turner angles between −45° and 45° represent regions where the stratification is stably stratified in both Θ and S_A. Turner angles greater than 90° or less than −90° characterize a statically unstable water column in which $N^2 < 0$. As a check on the calculation of the Turner angle, note that $R_\rho = -\tan\left(Tu + 45°\right)$. The Turner angle and the stability ratio are available in the GSW Oceanographic Toolbox from the function **gsw_Turner_Rsubrho**.