

Notes on the GSW library function gsw_Hill_ratio_at_SP2(t)

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This library function, **gsw_Hill_ratio_at_SP2(t)**, calculates the ratio by which the Hill *et al.* (1986) formula for Practical Salinity differs from that of the Practical Salinity Scale of 1978 (PSS-78, (Unesco (1981, 1983)) at a Practical Salinity of 2 and at the given input temperature t ($^{\circ}\text{C}$, ITS-90).

Practical Salinity (SP) is calculated in terms of the conductivity ratio (R_t) defined as (see Eqn. (E.2.1) of IOC *et al.* (2010))

$$R_t = \frac{C(S_p, t_{68}, 0)}{C(35, t_{68}, 0)},$$

being the ratio of two conductivity values, one from a general seawater sample and the other being a sample of Standard SeaWater (having a Practical Salinity of exactly 35), both being measured at the same temperature. Practical Salinity S_p is then calculated from the PSS-78 expression, Eqn. (E.2.6) of IOC *et al.* (2010), repeated here

$$S_p = \sum_{i=0}^5 a_i (R_t)^{i/2} + \frac{(t_{68} / ^{\circ}\text{C} - 15)}{[1 + k(t_{68} / ^{\circ}\text{C} - 15)]} \sum_{i=0}^5 b_i (R_t)^{i/2}. \quad (\text{E.2.6})$$

This formula is only valid when the resulting Practical Salinity is between 2 and 42. When Practical Salinity is less than 2 the GSW Oceanographic Toolbox uses a modified form of the Hill *et al.* (1986) expression for Practical Salinity. The output of the present function **gsw_Hill_ratio_at_SP2(t)** provides the multiplicative modification to other functions in the GSW Toolbox.

The first step in this library function **gsw_Hill_ratio_at_SP2(t)** is to calculate the t_{68} temperature from the t_{90} input temperature using Eqn. (A.1.3) of IOC *et al.* (2010), repeated here

$$(t_{68} / ^{\circ}\text{C}) = 1.00024 (t_{90} / ^{\circ}\text{C}). \quad (\text{A.1.3})$$

The PSS-78 expression Eqn. (E.2.6) is then solved for R_t at the known value of S_p of 2 using a modified Newton-Raphson iterative technique. With these values of R_t and t_{68} , the Hill *et al.* (1986) expression for Practical Salinity,

$$S_p = \sum_{i=0}^5 a_i (R_t)^{i/2} + \frac{(t_{68} / ^{\circ}\text{C} - 15)}{[1 + k(t_{68} / ^{\circ}\text{C} - 15)]} \sum_{i=0}^5 b_i (R_t)^{i/2} - \frac{a_0}{(1 + 600R_t + 160000(R_t)^2)} - \frac{(t_{68} / ^{\circ}\text{C} - 15)}{[1 + k(t_{68} / ^{\circ}\text{C} - 15)]} \frac{b_0}{(1 + 10(R_t)^{i/2} + 100R_t + 1000(R_t)^{3/2})},$$

is evaluated, and the ratio of 2 to this value of Practical Salinity is the output of this function, **gsw_Hill_ratio_at_SP2**. This ratio is used to modify the Hill *et al.* (1986) expression for Practical Salinity in several of the GSW functions. For example, this ratio is used in **gsw_SP_from_C** so that the Practical Salinity output of this function is a continuous function of conductivity when Practical Salinity transitions through the value 2. Note that the first line of the Hill *et al.* (1986) equation above is the PSS-78 expression and the second line contains the two correction terms of Hill *et al.* (1986) where a_0 and b_0 are the constants $a_0 = 0.008$ and $b_0 = 0.0005$ of PSS-78. .

References

- Hill, K. D., T. M. Dauphinee and D. J. Woods, 1986: The extension of the Practical Salinity Scale 1978 to low salinities, *IEEE J. Oceanic Eng.*, **11**, 109–112.
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