# Notes on the function gsw_CT_first_derivatives(SA,pt)

This function, **gsw_CT_first_derivatives**(SA,pt), evaluates the first derivatives of Conservative Temperature with respect to potential temperature and Absolute Salinity, as given in Eqn. (A.12.3a,b) in the TEOS-10 Manual (IOC *et al*. (2010)), repeated here.

$$\Theta_{\theta}\big|_{S_A} = c_p\left(S_A,\theta,0\right)/c_p^0, \qquad \Theta_{S_A}\big|_{\theta} = \left[\mu\left(S_A,\theta,0\right) - \left(T_0+\theta\right)\mu_T\left(S_A,\theta,0\right)\right]/c_p^0. \qquad \text{(A.12.3)}$$

Written in terms of the TEOS-10 Gibbs function these expressions become

$$\Theta_{\theta}\big|_{S_A} = -\left(T_0+\theta\right)g_{TT}\left(S_A,\theta,0\right)/c_p^0, \qquad (1)$$

and

$$\Theta_{S_A}\big|_{\theta} = \left[g_{S_A}\left(S_A,\theta,0\right) - \left(T_0+\theta\right)g_{S_A T}\left(S_A,\theta,0\right)\right]/c_p^0. \qquad (2)$$

This function uses the full TEOS-10 Gibbs function $g\left(S_A,t,p\right)$ of IOC *et al*. (2010), being the sum of the IAPWS-09 and IAPWS-08 Gibbs functions. Both the terms $g_{S_A}\left(S_A,\theta,0\right)$ and $\left(T_0+\theta\right)g_{S_A T}\left(S_A,\theta,0\right)$ in the square bracket in Eqn. (2) contain logarithmic singularities in the square root of Absolute Salinity, but these singularities exactly cancel in the square bracket. Hence, in the **gsw_CT_first_derivatives**(SA,pt) code this square bracket is not simply calculated by calling the appropriate derivatives of the Gibbs function, but rather, the polynomials representing both $g_{S_A}\left(S_A,\theta,0\right)$ and $g_{S_A T}\left(S_A,\theta,0\right)$ are incorporated into the code, and the logarithm terms are deliberately excluded. In this way, this function can be used with the input value of $S_A$ being exactly zero.

References


IAPWS, 2008: Release on the IAPWS Formulation 2008 for the Thermodynamic Properties of Seawater. The International Association for the Properties of Water and Steam. Berlin, Germany, September 2008, available from www.iapws.org. This Release is referred to in the text as **IAPWS-08**.

IAPWS, 2009: Supplementary Release on a Computationally Efficient Thermodynamic Formulation for Liquid Water for Oceanographic Use. The International Association for the Properties of Water and Steam. Doorwerth, The Netherlands, September 2009, available from http://www.iapws.org. This Release is referred to in the text as **IAPWS-09**.

IOC, SCOR and IAPSO, 2010: *The international thermodynamic equation of seawater – 2010: Calculation and use of thermodynamic properties*. Intergovernmental Oceanographic Commission, Manuals and Guides No. 56, UNESCO (English), 196 pp. Available from http://www.TEOS-10.org


Here follows appendix A.12 of the TEOS-10 Manual (IOC *et al*., 2010).

## A.12 Differential relationships between $\eta$, $\theta$, $\Theta$ and $S_\mathrm{A}$

Evaluating the fundamental thermodynamic relation in the forms (A.11.6) and (A.11.12) and using the four boxed equations in appendix A.11, we find the relations

$$\begin{aligned}(T_0+t)\,\mathrm{d}\eta+\mu(p)\,\mathrm{d}S_\mathrm{A} &= \frac{(T_0+t)}{(T_0+\theta)}\,c_p(0)\,\mathrm{d}\theta + \left[\mu(p)-(T_0+t)\,\mu_T(0)\right]\mathrm{d}S_\mathrm{A} \\ &= \frac{(T_0+t)}{(T_0+\theta)}\,c_p^0\,\mathrm{d}\Theta + \left[\mu(p)-\frac{(T_0+t)}{(T_0+\theta)}\mu(0)\right]\mathrm{d}S_\mathrm{A}\ .\end{aligned} \tag{A.12.1}$$

The quantity $\mu(p)\,\mathrm{d}S_\mathrm{A}$ is now subtracted from each of these three expressions and the whole equation is then multiplied by $(T_0+\theta)/(T_0+t)$ obtaining

$$(T_0+\theta)\,\mathrm{d}\eta \;=\; c_p(0)\,\mathrm{d}\theta-(T_0+\theta)\,\mu_T(0)\,\mathrm{d}S_\mathrm{A} \;=\; c_p^0\,\mathrm{d}\Theta-\mu(0)\,\mathrm{d}S_\mathrm{A}\ . \tag{A.12.2}$$

From this follows all the following partial derivatives between $\eta$, $\theta$, $\Theta$ and $S_\mathrm{A}$,

$$\Theta_\theta\big|_{S_\mathrm{A}} = c_p(S_\mathrm{A},\theta,0)/c_p^0, \qquad \Theta_{S_\mathrm{A}}\big|_\theta = \left[\mu(S_\mathrm{A},\theta,0)-(T_0+\theta)\,\mu_T(S_\mathrm{A},\theta,0)\right]/c_p^0, \tag{A.12.3}$$

$$\Theta_\eta\big|_{S_\mathrm{A}} = (T_0+\theta)/c_p^0, \qquad \Theta_{S_\mathrm{A}}\big|_\eta = \mu(S_\mathrm{A},\theta,0)/c_p^0, \tag{A.12.4}$$

$$\theta_\eta\big|_{S_\mathrm{A}} = (T_0+\theta)/c_p(S_\mathrm{A},\theta,0), \qquad \theta_{S_\mathrm{A}}\big|_\eta = (T_0+\theta)\,\mu_T(S_\mathrm{A},\theta,0)/c_p(S_\mathrm{A},\theta,0), \tag{A.12.5}$$

$$\theta_\Theta\big|_{S_\mathrm{A}} = c_p^0/c_p(S_\mathrm{A},\theta,0), \quad \theta_{S_\mathrm{A}}\big|_\Theta = -\left[\mu(S_\mathrm{A},\theta,0)-(T_0+\theta)\,\mu_T(S_\mathrm{A},\theta,0)\right]/c_p(S_\mathrm{A},\theta,0), \tag{A.12.6}$$

$$\eta_\theta\big|_{S_\mathrm{A}} = c_p(S_\mathrm{A},\theta,0)/(T_0+\theta), \qquad \eta_{S_\mathrm{A}}\big|_\theta = -\mu_T(S_\mathrm{A},\theta,0), \tag{A.12.7}$$

$$\eta_\Theta\big|_{S_\mathrm{A}} = c_p^0/(T_0+\theta), \qquad \eta_{S_\mathrm{A}}\big|_\Theta = -\mu(S_\mathrm{A},\theta,0)/(T_0+\theta). \tag{A.12.8}$$

The three second order derivatives of $\hat{\eta}(S_\mathrm{A},\Theta)$ are listed in Eqns. (P.14) and (P.15) of appendix P. The corresponding derivatives of $\hat{\theta}(S_\mathrm{A},\Theta)$, namely $\hat{\theta}_\Theta$, $\hat{\theta}_{S_\mathrm{A}}$, $\hat{\theta}_{\Theta\Theta}$, $\hat{\theta}_{S_\mathrm{A}\Theta}$ and $\hat{\theta}_{S_\mathrm{A}S_\mathrm{A}}$ can also be derived using Eqn. (P.13), obtaining

$$\hat{\theta}_\Theta = \frac{1}{\tilde{\Theta}_\theta}, \quad \hat{\theta}_{S_\mathrm{A}} = -\frac{\tilde{\Theta}_{S_\mathrm{A}}}{\tilde{\Theta}_\theta}, \quad \hat{\theta}_{\Theta\Theta} = -\frac{\tilde{\Theta}_{\theta\theta}}{\left(\tilde{\Theta}_\theta\right)^3}, \quad \hat{\theta}_{S_\mathrm{A}\Theta} = -\frac{\tilde{\Theta}_{\theta S_\mathrm{A}}}{\left(\tilde{\Theta}_\theta\right)^2} + \frac{\tilde{\Theta}_{S_\mathrm{A}}\tilde{\Theta}_{\theta\theta}}{\left(\tilde{\Theta}_\theta\right)^3}, \tag{A.12.9a,b,c,d}$$

$$\text{and}\quad \hat{\theta}_{S_\mathrm{A}S_\mathrm{A}} = -\frac{\tilde{\Theta}_{S_\mathrm{A}S_\mathrm{A}}}{\tilde{\Theta}_\theta} + 2\,\frac{\tilde{\Theta}_{S_\mathrm{A}}}{\tilde{\Theta}_\theta}\,\frac{\tilde{\Theta}_{\theta S_\mathrm{A}}}{\tilde{\Theta}_\theta} - \left(\frac{\tilde{\Theta}_{S_\mathrm{A}}}{\tilde{\Theta}_\theta}\right)^2\frac{\tilde{\Theta}_{\theta\theta}}{\tilde{\Theta}_\theta}, \tag{A.12.10}$$

in terms of the partial derivatives $\tilde{\Theta}_\theta$, $\tilde{\Theta}_{S_\mathrm{A}}$, $\tilde{\Theta}_{\theta\theta}$, $\tilde{\Theta}_{\theta S_\mathrm{A}}$ and $\tilde{\Theta}_{S_\mathrm{A}S_\mathrm{A}}$ which can be obtained by differentiating the polynomial $\tilde{\Theta}(S_\mathrm{A},\theta)$ from the TEOS-10 Gibbs function.